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# LINEAR OPERATION OF DISCRETE SIGNAL TRACKING

The idea of discrete signal "tracking" action was put into practice many years ago. So-called dot recorders are best examples of its application. The dynamics of the dot-recorder measuring mechanism can be linear and nonlinear. In both cases the recorder mechanisms are stable and their properties can be expressed by static gain (linear case) or certain static characteristics (nonlinear case). One can formulate two crucial questions: are we able to formulate the conditions for acceptable realization of discrete "tracking" action, and, if yes, how big are the tracking action errors if defined conditions are fulfilled. The paper answers both questions in the case of linear dynamics of the recorder mechanism.

Keywords: discrete 'tracking' action, conditions and accuracy of discrete 'tracking' action

## 1. INTRODUCTION

Let us consider that a linear measuring mechanism is excited by an input signal $x(t)$ and its response $Y(t)$ to $x(t)$ is described by the system of differential equations:

$$\begin{gathered}\sum_{i=0}^{m} a_i \frac{d^i y}{dt^i} = x(t) \\ Y(t) = \sum_{j=0}^{r} b_j \cdot \frac{d^j y}{dt^j}, j < m\end{gathered} \tag{1}$$

It is obvious that system (1) can be represented by the transfer function :

$$K(s) = \sum_{j=0}^{r} b_j \cdot s^j \left( \sum_{i=0}^{m} a_i \cdot s^i \right)^{-1} \tag{2}$$

or referring responses: the step $h(t)$ and impulse $k(t)$. The discretization of response $Y(t)$ to signal $x(t)$ makes that periodically, for $t = nT_i$, where $i = 1, 2, 3, \ldots$, the readings of $Y(nT_i)$ are taken (or recorded). Directly after that operation some values of derivatives $y^{(i)}(\mathrm{n}T_i)$ are brought to zero although the duration times ($\Delta t$) of the considered "reset" states can be treated as negligible. We can postulate that the result of the dotting operation should be $Y(nT_i) \cong x(T_i)$, which means that errors:

$$D(n \cdot T_i) = Y(n \cdot T_i) - x(n \cdot T_i), \tag{3}$$

are not too big [6]. However, errors (3) have to be determined.

Initially let us assume that the operation of reading $Y(nT_i)$ brings to zero values for $y^{(i)}(nT_i)$, $i = 1, 2, \ldots, m$, exactly like it happens during the operation of classic dot-recorders [1, 2, 5]. Using the dependence:

$$\begin{gathered} f(t) \mathrel{\hat{=}} F(s) \\ f^{(i)}(t) \mathrel{\hat{=}} s^i \cdot F(s) - \sum_{k=1}^{i} f^{(k-1)}(0) \cdot s^{i-k} \end{gathered} \tag{4}$$

and the system of Eq. (1) as well as the above assumption, after a certain number of simple transformations, one obtains the following result:

$$Y(s) = x(s) \cdot K(s) + \frac{b_0}{s} \cdot y_0 - \frac{y_0}{s} \cdot a_0 \cdot K(s), \tag{5}$$

where $x(s) \mathrel{\hat{=}} x(t)$, whereas $y_0$ denotes the value $y(n \cdot T_i) = \dfrac{Y(n \cdot T_i)}{b_0}$, if time range $nT_i \le t \le nT_i + T_i$ is taken into account. If $a_0 = 1$ we can transform Eq. (5) to the form:

$$Y(s) = b_0 \cdot \left\{ x(s) \cdot \frac{K(s)}{b_0} + \frac{y_0}{s} \cdot \left( 1 - \frac{K(s)}{b_0} \right) \right\}. \tag{6}$$

Hence :

$$Y(t) = b_0 \cdot \left\{ \int_0^t x(v) \cdot k_1(t-v) \cdot dv + \frac{Y(0)}{b_0} \cdot [1 - h_1(t)] \right\}, \tag{7}$$

where $b_0$ can be treated as a certain coefficient scaling the amplitude of $Y(t)$, $t$ belongs to the time range $nT_i \le t \le nT_i + T_i$ and step as well as impulse characteristics ($h_1(t)$ and $k_1(t)$ respectively) refer to transfer function $\dfrac{K(s)}{b_0}$ for $a_0 = 1$. Therefore, for $t = nT_i + T_i$ one obtains:

$$Y(n \cdot T_i + T_i) = Y(n \cdot T_i) \cdot [1 - h_1(T_i)] + b_0 \cdot \int_0^{T_i} x(n \cdot T_i + t - v) \cdot k_1(v) \cdot dv. \tag{8}$$

Furthermore, the above formula yields the stability condition of operation (8), which can be written in the form [5]:

$$0 < h_1 (T_i) < 2. \tag{9}$$

If we assume additionally that the operation of reading very quickly (inside of interval ($\Delta t$)) brings to zero the value $y(nT_i)$, then formula (5) immediately yields:

$$Y (n \cdot T_i + T_i) = \int_0^{T_i} x (n \cdot T_i + t - v) \cdot k (v) \cdot dv, \tag{10}$$

where $k(v)$ refers now to transfer function (2), for any parameters $a_i$, $b_j$.
The signal $x(t)$ can be expressed in a "specific" form [3, 4, 5, 6,]:

$$x (n \cdot T_i + t) = x (n \cdot T_i) + [x (n \cdot T_i + T_i) - x (n \cdot T_i)] \cdot \frac{t}{T_i} + \sum_{p=1}^{r} A (n \cdot T_i, p) \cdot \sin \frac{\pi \cdot t \cdot p}{T_i}. \tag{11}$$

The representation (11) remains in force for $0 \le t \le T_i$. The Eq. (11) can represent discontinuities in moments $nT_i$, $n = 0, 1, 2, \ldots$ It is obvious that values of amplitudes $A(nT_i, p)$ are different for each $n$. Calculating the integral (10) we obtain [5]:

$$\begin{aligned} c \int_0^{T_i} x (n \cdot T_i + t - v) \cdot k (v) \cdot dv &= H (T_i) \cdot [x (n \cdot T_i + T_i) - x (n \cdot T_i)] + \\ &+ x (n \cdot T_i) \cdot h (T_i) + \sum_{p=1}^{r} A (n \cdot T_i, p) \cdot S (p, T_i), \end{aligned} \tag{12}$$

where

$$H (T_i) = \frac{1}{T_i} \cdot \int_0^{T_i} h (t) \cdot dt, \quad S (p, T_i) = \int_0^{T_i} \sin \frac{\pi \cdot p \cdot v}{T_i} \cdot k (t - v) \cdot dv. \tag{13}$$

For the considered time interval $0 \le t \le T_i$ and "smooth" signal $x(t)$ is $A(nT_i, p) = 0$ and $x(n \cdot T_i + T_i) = x(n \cdot T_i) + T_i \cdot x^{(1)}(n \cdot T_i)$. These properties allow to rewrite the Eq. (8) in the form:

$$\begin{aligned} Y (n \cdot T_i + T_i) &= Y (n \cdot T_i) \cdot [1 - h_i (T_i)] + H_1 (T_i) \cdot [x (n \cdot T_i + T_i) - x (n \cdot T_i)] + \\ &+ x (n \cdot T_i) \cdot h_i (T_i) + \sum_{p=1}^{r} A (n \cdot T_i, p) \cdot S_1 (p, T_i) \quad . \end{aligned}$$

Then, after simple transformations, we obtain:

$$D(n \cdot T_i + T_i) = D(n \cdot T_i) \cdot [1 - h_1(T_i)] - x^{(1)}(n \cdot T_i) \cdot T_i \cdot [1 - H_1(T_i)] + \\ + \sum_{p=1}^{r} A(n \cdot T_i, p) \cdot S_1(p, T_i) \quad , \tag{14}$$

where $x^{(1)}(nT_i)$ denotes the derivative of the "smooth" component of signal $x(t)$. If $h_1(T_i) = 1$ and "non-smooth" components of signal $x(t)$ are neglected, then an error of the tracking action is caused only by certain "substitutive" delay of signal $Y(t)$ in relation to $x(t)$. The considered delay is:

$$T_{01} = T_i \cdot [1 - H_1(T_i)] \tag{15}$$

and formula (10) can be rewritten in the form:

$$Y(n \cdot T_i + T_i) = H(T_i) \cdot [x(n \cdot T_i + T_i) - x(n \cdot T_i)] + x(n \cdot T_i) \cdot h(T_i) + \\ + \sum_{p=1}^{r} A(n \cdot T_i, p) \cdot S(p, T_i) \tag{16}$$

Hence, after transformations similar to those given above, we obtain:

$$D(n \cdot T_i + T_i) = x(n \cdot T_i + T_i) \cdot [H(T_i) - 1] - x(n \cdot T_i) \cdot [H(T_i) - h(T_i)] + \\ + \sum_{p=1}^{r} A(n \cdot T_i, p) \cdot S(p, T_i). \tag{17}$$

Additionally, for $h(T_i) = 1$ is:

$$D(n \cdot T_i + T_i) = -x^{(1)}(n \cdot T_i) \cdot T_0 + \sum_{p=1}^{r} A(n \cdot T_i, p) \cdot S(p, T_i), \tag{18}$$

where

$$T_0 = T_i \cdot [1 - H(T_i)] \, .$$

In order to assure the high accuracy of discrete tracking operation on signal $x(t)$ we have to choose such parameters of $K(s)$ that $a_0 = 1$, $h_1(T_i) = 1$ the delay $T_{01}$ is minimized and susceptibilities $S_1(p, T_i)$ are minimized as well. The given list of requirements applies to the first case among those considered above. The parameters of $K(s)$ chosen for assumptions referring to second case should minimize delay $T_0$ as well as susceptibility $S(p, T_i)$ and make that $h(T_i) = 1$. To avoid current overload of the measuring device we should fulfill the additional condition:

$$\sup_{0 \le t \le T_i} |S(p, t)| < 1 \tag{19}$$

The stability postulation for the mechanism can be treated as inessential. Similarly, the duration time of reading $\Delta t$ does not generate further requirements, if initial conditions are "efficiently" brought to zero. Taking the throughput of information into account one should postulate small $T_i$.

## 2. PARTICULAR CASES, IF $y^{(i)}(nT_i) = 0$ FOR $i \geq 1$

The "classical" dot-recorder with an oscillatory mechanism, described by the transfer function:

$$K(s) = \frac{\omega_0^2}{\omega_0^2 + 2 \cdot B \cdot \omega_0 s + s^2}, \tag{20}$$

where $\omega_0$ – pulsation of proper vibration, $B$ – damping coefficient, has been described many times [3, 4,]. Summarizing the published results, we can recognize as optimal the following values: $B = 0.65$ and $\omega_0 T_i = 7.13$ (then $\omega_0 T_i = 1.29$). On the other hand, for $\omega_0 T_i = 5$ and $B = 0.192$, one obtains $H_1(T_i) = 1$, i.e. $T_0 \omega_0 = 0$, but condition (19) is not fulfilled. Let us note that condition $h(T_i) = 1$ is fulfilled for significantly smaller products:

$$\omega_0 \cdot T_i = \frac{\pi + 2 \cdot arcsinB}{2 \cdot \sqrt{1 - B^2}} \tag{21}$$

and the stability of the measuring mechanism, which is guaranteed for $B > 0$, has not to be preserved under the above circumstances.

These circumstances allow to consider the operation with negative $B$. The curve $\omega_0 T_i(B)$ shown in Fig. 1 guarantees that condition $h_1(T_i = 1)$ is fulfilled. The "associated" curve $\omega_0 T_0(B)$ obtained for that case is shown as well. We can observe that small values of damping coefficient $B$ or even negative $B$ are more advantageous comparing them to value $B$ recognized as optimal. The simulation experiments confirm that condition (19) holds and sensitivities $S_1(p, T_i)$ for such "non-optimal" choice of $B$, $T_i$ are smaller or at least comparable in relation to the classic case, i.e. for $B = 0.65$, $\omega_0 T_i = 7.13$. The conclusions formulated above are supported by data gathered in Table 1 and curves shown in Fig. 2.

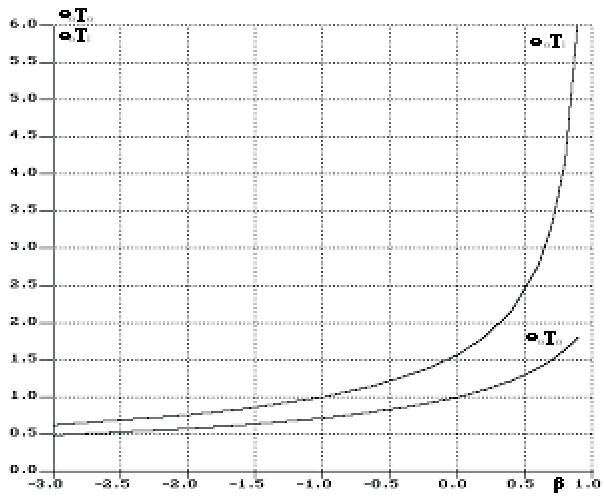


Fig. 1. Determination of those values of product $\omega_0 T_i$ which lead to fulfillment of condition $h_1(T_i) = 1$ and referring values of "substitute" delays $\omega_0 T_0$.

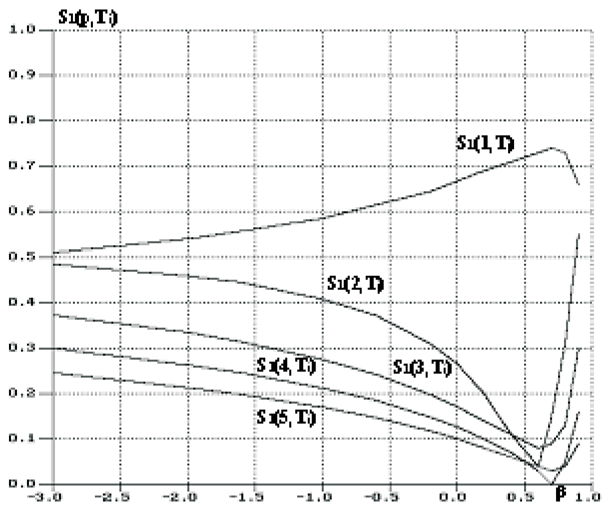


Fig. 2. The sensitivities $S_1(p, T_i)$ as functions of damping coefficient $\beta$ in case when $h_1(T_i) = 1$ holds.

Table 1. The values of sensitivities $S_1(p, T_i)$ for various damping coefficients $B$.

| $B$ | 0.65 | -3.00 | -2.50 | -2.00 | -1.50 | -1.00 | -0.50 | 0.00 | 0.50 | 0.90 |
|---|---|---|---|---|---|---|---|---|---|---|
| $\omega_0 T_i$ | 7.13 | 0.62 | 0.68 | 0.76 | 0.86 | 1.00 | 1.21 | 1.57 | 2.42 | 6.17 |
| $S(1, T_i)$ | 0.59 | 0.51 | 0.52 | 0.54 | 0.56 | 0.59 | 0.62 | 0.67 | 0.72 | 0.66 |
| $S(2, T_i)$ | 0.84 | 0.49 | 0.47 | 0.46 | 0.44 | 0.41 | 0.36 | 0.27 | 0.05 | 0.55 |
| $S(3, T_i)$ | 0.48 | 0.37 | 0.35 | 0.34 | 0.31 | 0.28 | 0.23 | 0.17 | 0.09 | 0.30 |
| $S(4, T_i)$ | 0.24 | 0.30 | 0.28 | 0.26 | 0.24 | 0.21 | 0.18 | 0.13 | 0.05 | 0.16 |
| $S(5, T_i)$ | 0.12 | 0.25 | 0.23 | 0.21 | 0.19 | 0.17 | 0.14 | 0.10 | 0.05 | 0.09 |
| ${}_0T_0$ | 1.29 | 0.49 | 0.53 | 0.58 | 0.64 | 0.72 | 0.83 | 1.00 | 1.30 | 1.81 |

The values $S_1(1, T_i)$ seem to be relatively high. Nevertheless we should take into consideration the possibility of minimization by shortening of $T_i$. The shorter $T_i$ the smaller components $A(\mathrm{n}T_i, 1)$. These decreases of the first amplitudes $A(\mathrm{n}T_i, 1)$ can be evaluated approximately – they are proportional to $T_i^2$. For $B = 0.6$ we can observe the distinct minimum of susceptibility functions referring to higher numbers $p$. Thus, the value $B = 0.6$ ought to be chosen for non-smooth signals. In such a case classical assumptions for the choice of value $T_i$ are not recommended – the proper choice of $T_i$ should be done using (21). On the contrary, small positive values $B$ or even negative ones are better for smooth signals.

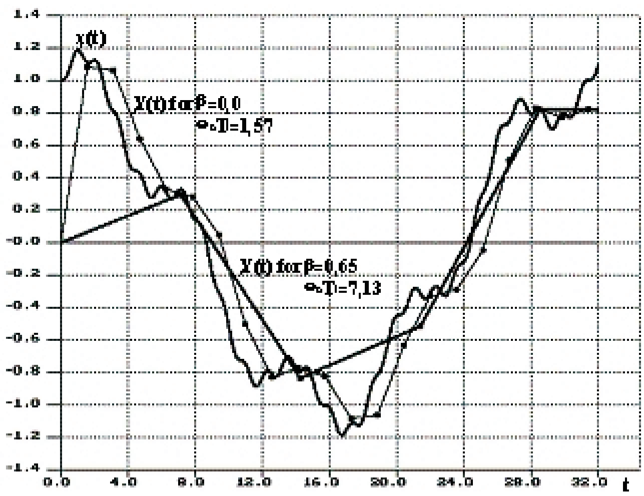


Fig. 3. The input signal $x(t)$ and results of interpolation of signal $Y(t)$ by broken lines for two cases, i.e. for $\beta = 0.65$, $\omega_0 T_i = 7{,}13$ and $\beta = 0.0$, $\omega_0 T_i = 1.57$.

The input signal $x(t)$ and points joined by a broken line representing results of tracking action for input $x(t)$ are shown in Fig. 3. The results in Fig. 3 have been obtained for two sets of parameters: $\omega_0 = 1s^{-1}$, $B = 0.65$, $T_i = 7.13s$ and $\omega_0 = 1s^{-1}$,

$B = 0$, $T_i = 1.57s$. Considering the result $Y(t)$ referring to the second set of parameters we can observe that the delay of the "smooth" component of signal $x(t)$ is about 1s and the obtained effect is more advantageous than in the case of classic recording.

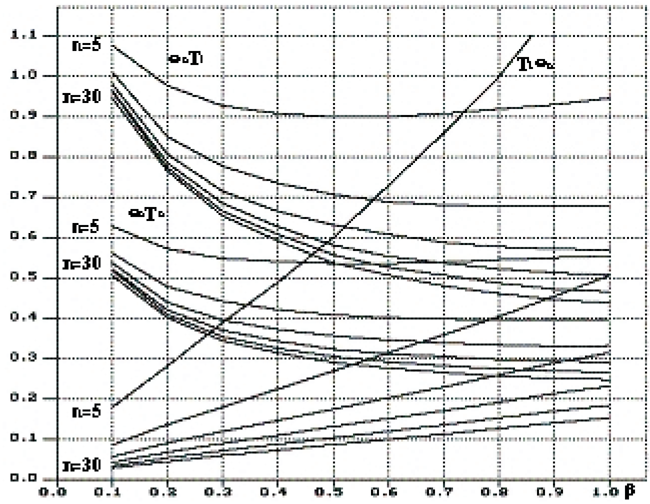

Fig. 4. The products $\omega_0 T_i$, $\omega_0 T_0$, $\omega - 0T_2$ as functions of damping coefficient $\beta$ for following n: 5, 10, 15, 20, 25, 30. The condition $h_1(\mathrm{T}_i)$ has been fulfiled.

It is worth an additional explanation that negative $B$ can be availed on the basis of the mechanism of a moving-coil meter by equipping it with an extra coil. Then the mechanism should be driven by the sum of $x(t)$ and a supplementary signal which represents a suitably amplified signal induced in the attached extra coil. Due to the described modification the whole system is equipped with an additional component (corrector) [6]. The corrector can be applied for other purposes as well. Let us assume that the corrector transfer function is:

$$K_k(s) = \frac{1 + sT_1}{1 + sT_2}. \tag{22}$$

Then the resultant transfer function of the system composed of a corrector and the measuring mechanism is represented by the product of transfer functions (20) and (22). For given $B$ and $\omega_0$ one can chose parameters $T_1$ and $T_2$ aiming at fulfillment of condition $h_1(T_i) = 1$. After that a suitable value of $T_0$ can be determined. The referring simulations have been done for several completing assumptions. It has been assumed that $T_1/T_2 = n$. The recommended choice of $T_2$ should assure that the maximum of $h_1(t)$ is smaller than $h_1(t)_{max} = 2$ but practically it should be almost equal to this value. Fulfilling all conditions mentioned above one obtains curves $\omega_0 T_i(B, n)$, $\omega_0 T_0(B, n)$ and $\omega_0 T_2(B, n)$ shown in Fig. 4. The referring curves representing sensitivities $S_1(p, T_i)$ are shown in Fig. 5. The changes of $S_1(p, T_i)$ do not depend heavily on $n$ and $B$. It means that the choice of $T_i$ and $T_0$ is crucial.

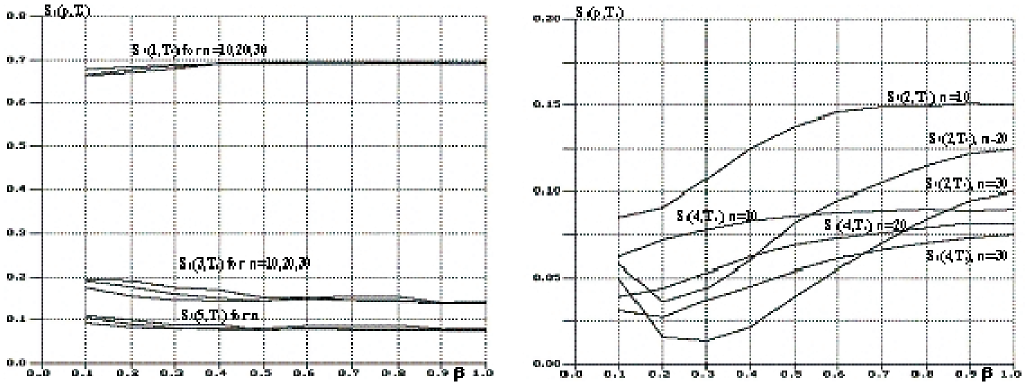

Fig. 5. The sensitivities $S_1(p, T_i)$ as functions of damping coefficient $\beta$ and $n = T_1/T_2$.

The curves in Fig. 6 represent respectively: $x(t)$ – input signal, $Y_1(t)$ – result of continuous recording for ${}_0 = 1s^{-1}$ and $B = 0.65$, $Y_2(t)$ – response of the system with discrete recording for $T_i = 1.7$, $\omega_0 = 1s^{-1}$, $B = 0$, $T_0 = 1s$, $Y_3(t)$ – response of the system with discrete recording for $T_i = 0.44$, $\omega_0 = 1s^{-1}$, $n = 30$, $T_2 = 0.15$, $B = 1$, $T_0 = 0.25s$. The points of discrete recording refer to points where curves $Y_2(t)$, $Y_3(t)$ are “broken” (the recording is done in discontinuity points of derivatives of $Y_2(t)$ and $Y_3(t)$). The “heoretical” advantage caused by application of the corrector seems to be undoubted, however some difficulties can appear if practical use of the corrector is considered. The possibility of big values of corrector output signal appearing for big $n$ can be recognized as the main cause of these technical difficulties. Of course, the mentioned technical inconvenience does not cause trouble during the simulation of a real experiment. The omission of condition $h_1(t)_{max} = 2$ and extension of the range of changes of $B$ and $n$ allow to continue the process of further diminishing of $T_i$ and $T_0$.

## 3. PARTICULAR CASES, WHEN $y^{(i)}(nT_i) = 0$ FOR $i = 0, 1, \ldots, m$

Now we have no restrictions imposed on values of parameters of the transfer function (2) and formula (10) is still in force. This allows to consider a wide class of measuring devices, for example, devices with static gains greater than 1, those with dynamics similar to certain “standards” (having properties of integrator, differentiator or other “standards”), etc. Considering an extremely simple case, if

$$K(s) = \frac{k}{1 + s \cdot T}, \tag{23}$$

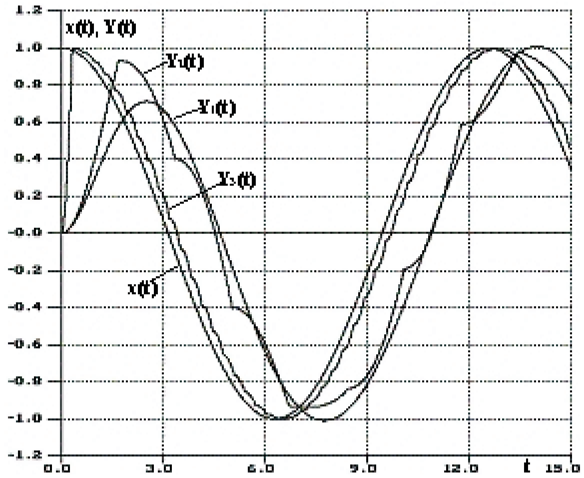


Fig. 6. Input signal $x(t)$, the result of continuous-tracking action $Y_1(t)$ and results of discrete tracking actions $Y_2(t)$, $Y_3(t)$ obtained under conditions described in the paper.

we obtain:

$$h(t) = k \cdot \left(1 - e^{-t/T}\right), \quad T_i = -T \cdot \ln\left(1 - \frac{1}{k}\right), \tag{24}$$

$$H(T_i) = k \cdot \left\{1 - \frac{T}{T_i} \cdot \left(1 - e^{-T_i/T}\right)\right\} = \frac{1}{\ln\left(1 - \frac{1}{k}\right) + k} \tag{25}$$

and

$$S(p, T_i) = \frac{k \cdot \pi \cdot p \cdot \left(\cos(\pi \cdot p) + \frac{1}{k} - 1\right)}{\left[1 + \left[\frac{\pi \cdot p}{\ln\left(1 - \frac{1}{k}\right)}\right]^2\right] \cdot \ln\left(1 - \frac{1}{k}\right)}. \tag{26}$$

The curves representing quotients $T_i/T$, $T_0/_T$ and susceptibility $S(p, T_i)$ as functions of gain $k$ are shown in Fig. 7. We cannot obtain $T_0 = 0$, but for sufficiently big $k$ one can obtain suitably small values $T_i$, $T_0$ conserving small values of sensitivities $S(p, T_i)$. For example, if $k = 5.5$, $T = 5s$ then $T_i = 1s$, $T_0 0.5s$ and $S(1, T_i) = 0.63$, $S(2, \mathrm{T}_i) = 0.0\ldots$, $S(3, T_i) = 0.21$, $S(4, T_i) = -0.016$, $S(5, T_i) = 0.014$. The input signal $x(t)$, response of the system with discrete recording $Y_1 1(t)$ and the result of discrete recording of $Y_2(t)$ interpolated by means of a broken line are shown in Fig. 8a.

For a measuring device with integrating properties:

$$K(s) = \frac{1}{s \cdot T_c \cdot (1 + s \cdot T)} \tag{27}$$

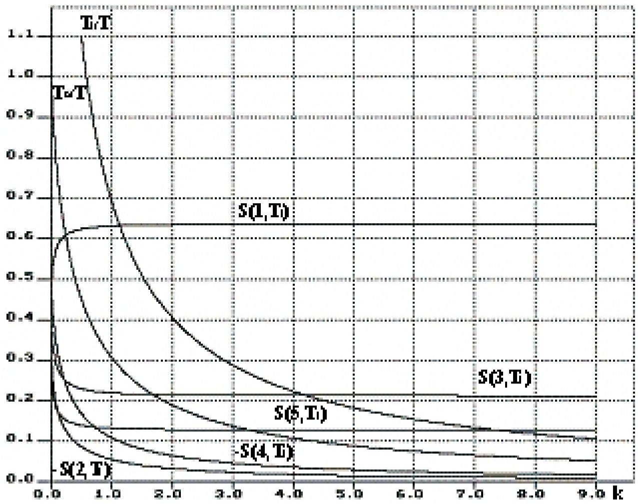


Fig. 7. The ratios $T_i/T$, $T_0/T$ and sensitivities $S(p, T_i)$ as functions of $k$ for $h(T_i) = 1$.

we obtain:

$$h(t) = \frac{T}{T_c} \cdot \left( \frac{t}{T} + e^{-\frac{t}{T}} - 1 \right)$$

and equation

$$\frac{T_i}{T} + e^{-T_i/T} = 1 + \frac{T_c}{T} = 1 + Q \tag{28}$$

Approximation of (28) yields

$$\frac{T_i}{T} \cong 1 + Q - 0.48 \cdot e^{-1 \cdot 107 \cdot Q}$$

or for big Q:

$$T_i \cong T_c \cdot \left(1 + \frac{1}{Q}\right), T_0 \cong \frac{1}{2} \cdot T_c \cdot \left(1 + \frac{1}{Q}\right)^2$$

$$S(p, T_c) \cong \frac{1}{\left[1 + \left(\frac{\pi \cdot p}{1+Q}\right)^2\right]} \cdot \left\{ \frac{\pi \cdot p}{Q \cdot (1+Q)} \cdot \left(1 - e^{-(1+Q)}\right) - \frac{\left(1 + \frac{1}{Q}\right)}{\pi \cdot p} \cdot [\cos(\pi \cdot \mathrm{p}) - 1] \right\}. \tag{29}$$

Now we can observe ratios $T_0/T$ similar to those obtained for measuring device dynamics given by the first-order inertia model (23). The input signal $x(t)$, response of the system with discrete recording $Y_1(t)$ and the result of discrete recording after

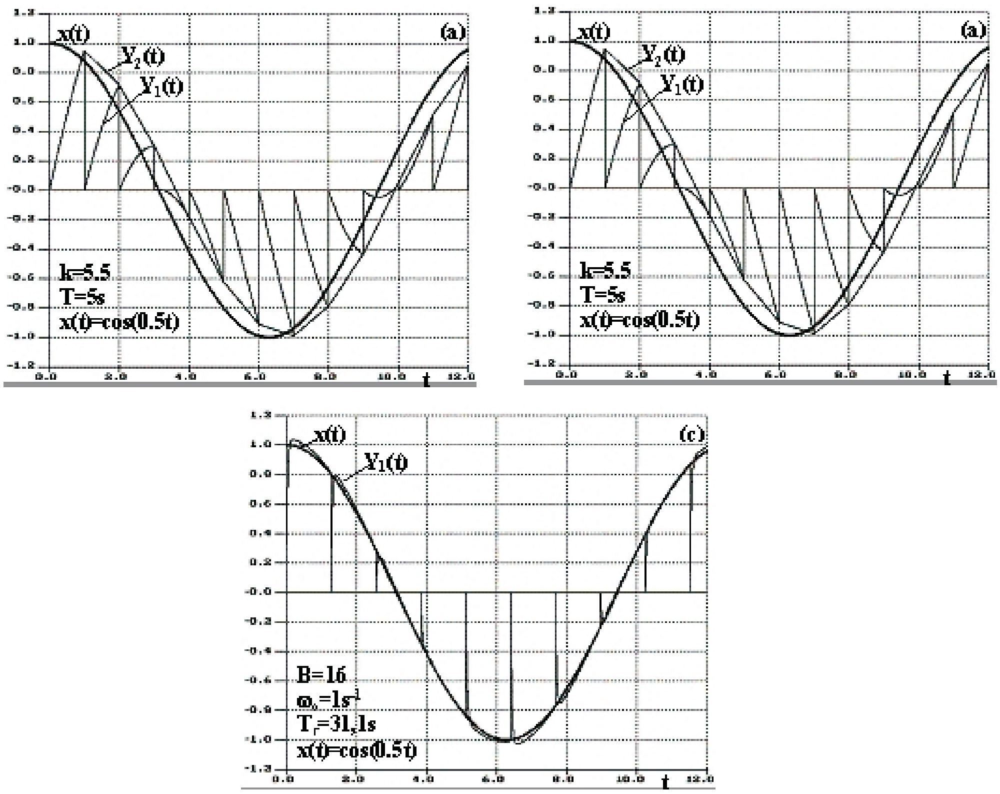


Fig. 8. a,b,c – exact explanations can be found in the paper.

interpolation with broken line are shown in Fig 8b. The presented results have been obtained for $T_c = 1s$, $Q = 5$.

For differentiating properties of the measuring device:

$$K(s) = \frac{s \cdot T_r \cdot \omega_0}{s^2 + 2 \cdot B \cdot \omega_0 \cdot s + \omega_0^2} \tag{30}$$

and given values $B$ and $\omega_0$ and sufficiently big $T_r$ one obtains two values $T_{i1}$, $T_{i2}$ yielding $h[T_i] = 1$. Putting $B = 1$ one can choose such $T_r$, that for $T_{i2}$ (larger component of pair $T_{i1}$, $T_{i2}$) is $H(T_{i2}) = 1$. Due to the considered choice the delay is eliminated completely. The curves representing product $T_r\omega_0$ as a function of those damping coefficients $B$ which make that $h(T_i) = 1$ and $H(T_{i2}) = 1$ are shown in Fig. 9. The products $T_{i1}\omega_0$ and $T_0\omega_0$ for $T_{i1}$ are presented in Fig. 9 too. The signal $x(t)$ as well as the response of the system with discrete recording for $T_{i2}$ (i.e. for $T$) defined by the following parameters $\omega_0 = 1s^{-1}$, $B = 16$, $T_r = 31.11s$, $t_{i2} = 1.28s$ are shown in Fig. 8c. It results from Fig. 9 that time $T_{i1}$ and accompanying delay $T_0$ are relatively "short" and

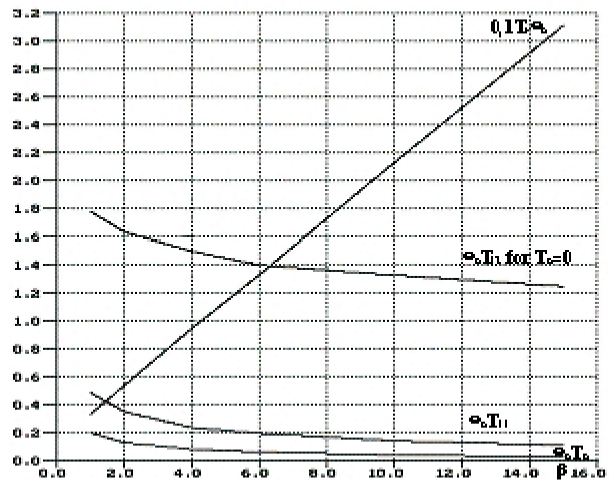


Fig. 9. The products $T_r\omega_0$ as functions of B which make that for $h(T_{i1}) = 1$, delay $T_0 \neq 0$ and for $h(T_{i2}) = 1$ the delay is $T_0 = 0$. The curves referring to those products and representing functions $T_{i1}(B)$ and $T_0(B)$ are shown as well.

the proposed choice of measuring device parameters can occur even more advantageous than in case of considerably "longer" time $T_{i2}$ and $T_0 = 0$. Necessary calculations (also aimed at determining the susceptibility $S(p, T_i)$) have to be done using a computer. That is why the formulation of analytical dependences seems to be a very difficult task. Calculations made for the considered assumptions yield: $S(1, T_{i1}) = 0.5$, $S(1, T_{i2}) = 0.07$, $S(2, T_{i1}) = -0.43$, $S(2, T_{i2}) = -0.17$, $S(3, T_{i1}) = 0.35$, $S(3, T_{i2}) = 0.26$, $S(4, T_{i1]}) = -0.27$, $S(4, T_{i2}) = -0.34$, $S(5, T_{i1}) = 0.22$, $S(5, T_{i2}) = 0.38$. We can expect that other values of the damping coefficient B yield similar values of sensitivities.

## 4. SUMMARY

The principles for the choice of parameters of the recorder in case of dot-recording based on an electromechanical measuring system (also in case of use of a corrector) should differ from those applied classically. On the other hand, if $h(T_i) = 1$, then linear operation of discrete tracking action in case of using the condition for zeroing of values $y^{(i)}(nT_i) = 0$ (for $i$ = 0, 1, 2,... $m$) allows to keep the tracking action errors at a reasonably small level, however this property becomes real only for high values of throughput of information (i.e. for "short" $T_i$). We must honestly admit that the problem of discrete tracking action in case of nonlinear dynamics seems to be still open.

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